Chapter 5: The Importance of Measuring Variability

- Measures of Central Tendency - Numbers that describe what is typical or "central" in a variable's distribution (e.g., mean, mode, median).
- Measures of Variability - Numbers that describe diversity or variability in a variable's distribution (e.g., range, variance, standard deviation).

Why is Variability important?

Example: Suppose you wanted to know how satisfied students are with their living arrangements and you found that the mean answer was "3" on a five point scale where:

1=very unsatisfied, 2=satisfied, 3=neutral, 4= satisfied, 5=very satisfied

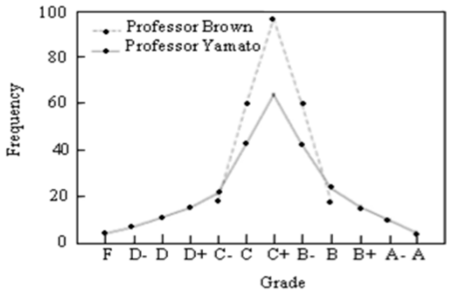
What would you conclude?

Would knowing the variability of the answers help you to understand how satisfied students are with their living arrangements?

Answer: It would help you to see whether the average score of "3" means that the majority of students are neutral about their living arrangements

or

that there is a split with students either feeling very satisfied (score of 5) or unsatisfied (score of 1) with their living arrangements (average of 1's and 5's = 3).



Another example.

The Range

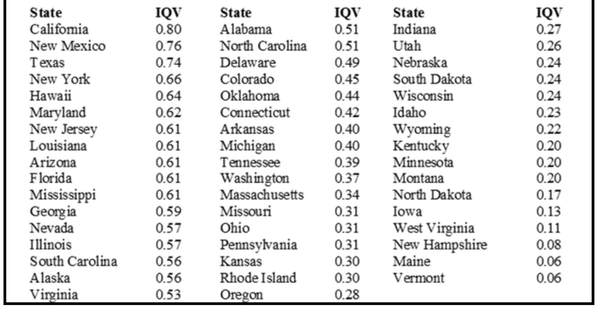
- Range – A measure of variation in interval-ratio variables.
- It is the difference between the highest (maximum) and the lowest (minimum) scores in the distribution.

Range = highest score - lowest score

What is the range for these IQV (diversity) scores?

(higher IQV scores means more diversity)

Steps to determine: subtract the lowest score ____from the highest _____to obtain the range of IQV scores_____.



State	IQV	State	IQV	State	IQV
California	0.80	Alabama	0.51	Indiana	0.27
New Mexico	0.76	North Carolina	0.51	Utah	0.26
Texas	0.74	Delaware	0.49	Nebraska	0.24
New York	0.66	Colorado	0.45	South Dakota	0.24
Hawaii	0.64	Oklahoma	0.44	Wisconsin	0.24
Maryland	0.62	Connecticut	0.42	Idaho	0.23
New Jersey	0.61	Arkansas	0.40	Wyoming	0.22
Louisiana	0.61	Michigan	0.40	Kentucky	0.20
Arizona	0.61	Tennessee	0.39	Minnesota	0.20
Florida	0.61	Washington	0.37	Montana	0.20
Mississippi	0.61	Massachusetts	0.34	North Dakota	0.17
Georgia	0.59	Missouri	0.31	Iowa	0.13
Nevada	0.57	Ohio	0.31	West Virginia	0.11
Illinois	0.57	Pennsylvania	0.31	New Hampshire	0.08
South Carolina	0.56	Kansas	0.30	Maine	0.06
Alaska	0.56	Rhode Island	0.30	Vermont	0.06
Virginia	0.53	Oregon	0.28		

What is the range for these diversity scores?

Steps to determine: subtract the lowest score _.06___from the highest _____to obtain the range of IQV scores_____.

State	IQV	State	IQV	State	IQV
California	0.80	Alabama	0.51	Indiana	0.27
New Mexico	0.76	North Carolina	0.51	Utah	0.26
Texas	0.74	Delaware	0.49	Nebraska	0.24
New York	0.66	Colorado	0.45	South Dakota	0.24
Hawaii	0.64	Oklahoma	0.44	Wisconsin	0.24
Maryland	0.62	Connecticut	0.42	Idaho	0.23
New Jersey	0.61	Arkansas	0.40	Wyoming	0.22
Louisiana	0.61	Michigan	0.40	Kentucky	0.20
Arizona	0.61	Tennessee	0.39	Minnesota	0.20
Florida	0.61	Washington	0.37	Montana	0.20
Mississippi	0.61	Massachusetts	0.34	North Dakota	0.17
Georgia	0.59	Missouri	0.31	Iowa	0.13
Nevada	0.57	Ohio	0.31	West Virginia	0.11
Illinois	0.57	Pennsylvania	0.31	New Hampshire	0.08
South Carolina	0.56	Kansas	0.30	Maine	0.06
Alaska	0.56	Rhode Island	0.30	Vermont	0.06
Virginia	0.53	Oregon	0.28		

What is the range for these diversity scores?

Steps to determine: subtract the lowest score _.06___from the highest __.80___to obtain the range of IQV scores_____.

State	IQV	State	IQV	State	IQV
California	0.80	Alabama	0.51	Indiana	0.27
New Mexico	0.76	North Carolina	0.51	Utah	0.26
Texas	0.74	Delaware	0.49	Nebraska	0.24
New York	0.66	Colorado	0.45	South Dakota	0.24
Hawaii	0.64	Oklahoma	0.44	Wisconsin	0.24
Maryland	0.62	Connecticut	0.42	Idaho	0.23
New Jersey	0.61	Arkansas	0.40	Wyoming	0.22
Louisiana	0.61	Michigan	0.40	Kentucky	0.20
Arizona	0.61	Tennessee	0.39	Minnesota	0.20
Florida	0.61	Washington	0.37	Montana	0.20
Mississippi	0.61	Massachusetts	0.34	North Dakota	0.17
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What is the range for these diversity scores?

Steps to determine: subtract the lowest score _.06___from the highest __.80___to obtain the range of IQV scores__.74___.

State	IQV	State	IQV	State	IQV
California	0.80	Alabama	0.51	Indiana	0.27
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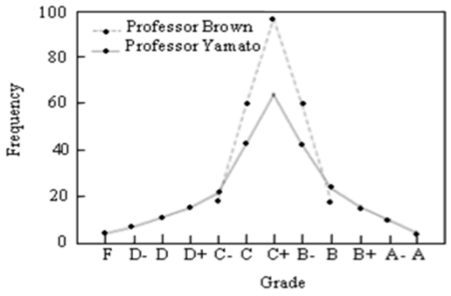


Another example.

Inter-quartile Range

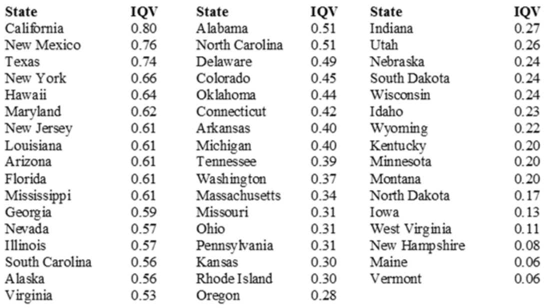
- Inter-quartile range (IQR) – The width of the middle 50 percent of the distribution.
- The IQR helps us to get a better picture of the variation in the data than the range because it focuses on the width of the middle 50% rather than extreme scores in the distribution.
- The shortcoming of the range is that an “outlying” case at the top or bottom can increase the range substantially.

Inter-quartile Range

- Inter-quartile range (IQR) – The width of the middle 50 percent of the distribution.
- It is defined as the difference between the lower and upper quartiles (Q1 and Q3.)
- IQR = q3 – q1

(e.g., 75th percentile – 25th percentile)

What is the IQR for these Diversity Scores?



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Virginia	0.53	Oregon	0.28		

(Steps are provided on the next slides)

What is the IQR for the Diversity Scores?

Steps to determine the IQR (Q3 – Q1):

1. Order the categories from highest to lowest (or vice versa)
2. To obtain Q1, begin by dividing N (total number of categories or states) by 4 (or alternatively multiply N by .25). This equals______?
3. We now know that Q1 falls between the 12th and 13th category or, in this case, states.
4. To find the exact number for Q1, determine the midpoint between the 12th and 13th states or between .59 and .57)
5. Q1 = ____

What is the IQR for the Diversity Scores?

Steps to determine the IQR (Q3 – Q1):

1. Order the categories from highest to lowest (or vice versa)
2. To obtain Q1, begin by dividing N (total number of categories or states) by 4 (or alternatively multiply N by .25). This equals___12.5___?
3. We now know that Q1 falls between the 12th and 13th category or, in this case, states.
4. To find the exact number for Q1, determine the midpoint between the 12th and 13th states or between .59 and .57)
5. Q1 = ____

What is the IQR for the Diversity Scores?

Steps to determine the IQR (Q3 – Q1):

1. Order the categories from highest to lowest (or vice versa)
2. To obtain Q1, begin by dividing N (total number of categories or states (50)) by 4 (or alternatively multiply N by .25). This equals___12.5___?
3. We now know that Q1 falls between the 12th and 13th category or, in this case, states.
4. To find the exact number for Q1, determine the midpoint between the 12th and 13th states or between .59 and .57)
5. Q1 = __.58__

What is the IQR for the Diversity Scores?

Steps to determine the IQR (Q3 – Q1):

6. To obtain Q3, begin by multiplying 12.5 by 3 (or alternatively multiply N (50) by .75). This will give us____.
7. Based on this number, Q3 falls between the 37th and 38th states.
8. Determine the midpoint between these two states. This equals_____. This tells us that 50% of the cases fall between .58 and .24.
9. To obtain the IQR subtract Q3 from Q1 which equals______or the middle of the middle 50% of the cases.

What is the IQR for the Diversity Scores?

Steps to determine the IQR (Q3 – Q1):

6. To obtain Q3, begin by multiplying 12.5 by 3 (or alternatively multiply N (50) by .75). This will give us__37.5__.
7. Based on this number, Q3 falls between the 37th and 38th states.
8. Determine the midpoint between these two states. This equals_____. This tells us that 50% of the cases fall between .58 and .24.
9. To obtain the IQR subtract Q3 from Q1 which equals______or the middle of the middle 50% of the cases.

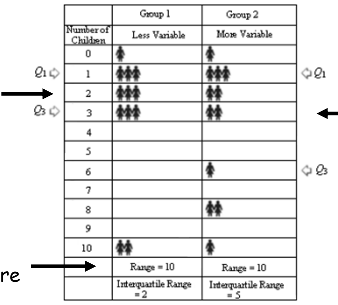
What is the IQR for the Diversity Scores?

Steps to determine the IQR (Q3 – Q1):

6. To obtain Q3, begin by multiplying 12.5 by 3 (or alternatively multiply N (50) by .75). This will give us__37.5__.
7. Based on this number, Q3 falls between the 37th and 38th states.
8. Determine the midpoint between these two states. This equals___.24__. This tells us that 50% of the cases fall between the IQR scores of .58 and .24.
9. To obtain the IQR subtract Q3 from Q1 which equals_.34____

The difference between the Range and IQR

These values fall together closely



	Group 1	Group 2
Number of Children	Less Variable	More Variable
0	🧍	🧍
1	🧍🧍🧍	🧍🧍🧍
2	🧍🧍🧍	🧍🧍
3	🧍🧍🧍	🧍🧍
4		
5		
6		🧍
7		
8		🧍🧍
9		
10	🧍🧍	🧍
	Range = 10	Range = 10
	Interquartile Range = 2	Interquartile Range = 5

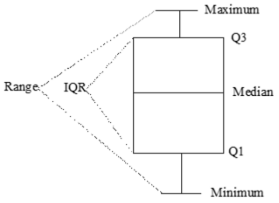
Shows greater variability

Yet the ranges are equal!

Importance of the IQR

The Box Plot

- The Box Plot is a **graphic device** that visually presents the following elements: the **range**, the **IQR**, the **median**, the **quartiles**, amount and direction of **skewness**, the **minimum** (lowest value,) and the **maximum** (highest value.)



Measures of Variability:

Shortcomings of the Range and IQR

- The range is based on only two categories (the highest and lowest)
- Likewise, only two categories are used to calculate the inter-quartile range.
- Neither allows us to know how much variation there is among all the categories.

Measures of Variability: the Variance

- The variance allows us to account for the total amount of variation.
- The variance is an important statistic that is used in most other sophisticated statistics. Therefore, it is important for you to give it particular attention.

Be sure to read the sections of the chapter on variability and standard deviation very carefully.

Determining the Variance in the "Percentage Increase" in the Nursing Home Population, 1980-1990

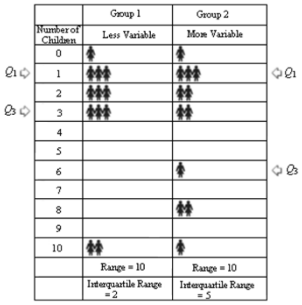
Nine Regions of U.S.	Percentage
Pacific	15.7
West North Central	16.2
New England	17.6
East North Central	23.2
West South Central	24.3
Middle Atlantic	28.5
East South Central	38.0
Mountain	47.9
South Atlantic	71.7

What statistics have we learned so far to describe the variation above?

Is there a lot of variation between the categories (regions of U.S.)?

Range, Inter-Quartile Range (IQR)
There appears to be a lot of variation between regions.

Comparing the Range and the Variance



	Number of Children	Group 1 Less Variable	Group 2 More Variable	
	0	👤	👤	
Q1 ⇨	1	👤👤👤	👤👤👤	⇦ Q1
	2	👤👤👤	👤👤	
Q3 ⇨	3	👤👤👤	👤👤	
	4			
	5			
	6		👤	⇦ Q3
	7			
	8		👤👤	
	9			
	10	👤👤	👤	
		Range = 10	Range = 10	
		Interquartile Range = 2	Interquartile Range = 5	

The range uses only two of the cases to determine/calculate the variability of a group.

We will see that the variance considers every single case prior to determining/calculating the degree of variance.

First Step in Calculating the Variance: Determine the "**Average**" score for the nine regions

Nine Regions of U.S.	Percentage
Pacific	15.7
West North Central	16.2
New England	17.6
East North Central	23.2
West South Central	24.3
Middle Atlantic	28.5
East South Central	38.0
Mountain	47.9
South Atlantic	71.7

The "average" percentage change in the Nursing Home (NH) Population, 1980-1990

Nine Regions of U.S.	Change in NHs (percent reported)
Pacific	15.7
West North Central	16.2
New England	17.6
East North Central	23.2
West South Central	24.3
Middle Atlantic	28.5
East South Central	38.0
Mountain	47.9
South Atlantic	71.7
$\sum$ Y =	283.1

Average "% increase"

mean = $\bar{Y} = \frac{\sum Y}{N}$ = 31.45

Determining the Variation in the Percentage Change in the Nursing Home Population, 1980-1990

Nine Regions of U.S.	Percentage	$Y - \bar{Y}$
Pacific	15.7	15.7 - 31.5 = -15.8
West North Central	16.2	16.2 - 31.5 = -15.3
New England	17.6	17.6 - 31.5 = -13.9
East North Central	23.2	23.2 - 31.5 = - 8.3
West South Central	24.3	24.3 - 31.5 = - 7.2
Middle Atlantic	28.5	28.5 - 31.5 = - 3.0
East South Central	38.0	38.0 - 31.5 = 6.5
Mountain	47.9	47.9 - 31.5 = 16.4
South Atlantic	71.7	71.7 - 31.5 = 40.2
(mean = 31.5)	$\sum$ Y = 283.1	$\sum (Y - \bar{Y}) = 0$

Next, we can determine the distance between (1) each region and (2) the average. This will show us how much each case (i.e., region) varies from the mean. Then, we can add up the variation scores for each region to get the "total" variation for all the cases (or regions in this example).

Percentage Change in the Nursing Home Population, 1980-1990

Nine Regions of U.S.	Percentage	$Y - \bar{Y}$
Pacific	15.7	15.7 - 31.5 = -15.8
West North Central	16.2	16.2 - 31.5 = -15.3
New England	17.6	17.6 - 31.5 = -13.9
East North Central	23.2	23.2 - 31.5 = - 8.3
West South Central	24.3	24.3 - 31.5 = - 7.2
Middle Atlantic	28.5	28.5 - 31.5 = - 3.0
East South Central	38.0	38.0 - 31.5 = 6.5
Mountain	47.9	47.9 - 31.5 = 16.4
South Atlantic	71.7	71.7 - 31.5 = 40.2
(mean = 31.5)	$\sum$ Y = 283.1	$\sum (Y - \bar{Y}) = 0$

Problem: when you add up the distances you end up with zero rather than the total variation from all the categories. Why is this?

Nine Regions of U.S.	Percentage	$Y - \bar{Y}$
Pacific	15.7	15.7 - 31.5 = -15.8
West North Central	16.2	16.2 - 31.5 = -15.3
New England	17.6	17.6 - 31.5 = -13.9
East North Central	23.2	23.2 - 31.5 = - 8.3
West South Central	24.3	24.3 - 31.5 = - 7.2
Middle Atlantic	28.5	28.5 - 31.5 = - 3.0
East South Central	38.0	38.0 - 31.5 = 6.5
Mountain	47.9	47.9 - 31.5 = 16.4
South Atlantic	71.7	71.7 - 31.5 = 40.2
(mean = 31.5)	$\sum$ Y = 283.1	Absolute value = 126.6

- One solution would be to add up the absolute values for each number (ignore the minus signs). In this example, the absolute value is 126.6. We would then divide by the number of regions or 9 to obtain the average variation, or 14.1. Unfortunately, absolute values are very difficult to work with mathematically when using more complex statistics such as regression analysis.
- Fortunately, there is an alternative to using the absolute values in order to get the average variance.

Percentage Change in the Nursing Home Population, 1980-1990

Nine Regions of U.S.	Percentage	$Y - \overline{Y}$	$(Y - \overline{Y})^2$ (squared deviations)
Pacific	15.7	15.7 - 31.5 = -15.8	249.64
West North Central	16.2	16.2 - 31.5 = -15.3	234.09
New England	17.6	17.6 - 31.5 = -13.9	193.21
East North Central	23.2	23.2 - 31.5 = - 8.3	68.89
West South Central	24.3	24.3 - 31.5 = - 7.2	51.84
Middle Atlantic	28.5	28.5 - 31.5 = - 3.0	9.00
East South Central	38.0	38.0 - 31.5 = 6.5	42.25
Mountain	47.9	47.9 - 31.5 = 16.4	268.96
South Atlantic	71.7	71.7 - 31.5 = 40.2	1616.04
(mean = 31.5)	$\sum \overline{Y}$ = 283.1		$\sum (Y - \overline{Y})^2 = 2733.92$

- The best solution is to square the differences before adding them up (when two negative numbers are multiplied the resulting product is a positive number). Then we can get the average squared variance and then un-square it to have the average variance (once un-squaring it, the number is called the "standard deviation" rather than the variance).

Measures of Variability: the Variance

The Variance is the average of the squared deviations (differences) from the mean.

Variance = $$s_Y^2 = \frac{\sum (Y - \overline{Y})^2}{N-1}$$

In our example we would take the sum of the squared differences (2733.92) and divide this number by the total number of cases minus one (9 – 1 = 8). This would give us _341.74___ or the variance for the Percent Increase in the Nursing Home population by region.

Measures of Variability: The Variance

To Sum Up:

The Variance is the average of the squared differences from the mean.

The Variance is a measure of variability for interval-ratio variables.

$$s_Y^2 = \frac{\sum (Y - \overline{Y})^2}{N-1}$$

Measures of Variability: Standard Deviation

- One problem with the variance is that the final number obtained is in a squared form

  (that is, we squared all the differences from the mean and so the final number is still "inflated" in this way making it difficult to interpret)

- One solution is to un-square the variance, that is, take the square root of the variance so that the number is no longer in a squared form (or "inflated") and it is back to its original form. The square root of the variance is called the Standard Deviation (or standard difference).

Measures of Variability: Standard Deviation

- To obtain the square root of the variance simply enter the number (variance) into your calculator and then push the square root button.

- If the variance is 341.74 the standard deviation would be ___18.49_____. This tells us that the average distance of a case (region) from the mean is 18.49.

- Thus, this standard deviation tells us that there is a lot of variation between the regions. (If the standard deviation had been 2, we would have concluded that the regions did not vary among themselves very much.)

In Sum

The Standard Deviation is a measure of variation for interval-ratio variables; it is equal to the square root of the variance.

$$s = \sqrt{s_Y^2} = \sqrt{\frac{\sum (Y - \overline{Y})^2}{N-1}}$$

Measures of Variability: Standard Deviation

(a look at what's to come in future chapters)

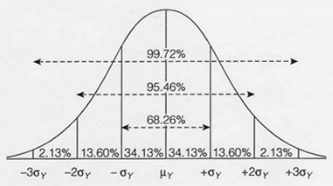
We will see later that when the data are "normally distributed" around the mean (produce a normal curve), 34% of the scores will be one standard deviation (that is, the average distance) above the mean and 34% will be one standard deviation (average distance) below the mean.

Scores are often "normally distributed" around the mean when a random sample has been used to obtain the scores or there are a large number of cases.

A Leap of Faith:

We can use the **normal curve** and **standard deviation** to determine the percentage of cases that are close to the mean as well as the percentage that are far from the mean.

Figure 10.3 **Percentages Under the Normal Curve**



Considerations for Choosing a Measure of Variability

- For ordinal variables, you can calculate the IQR (inter-quartile range.)
- For interval-ratio variables, you can use IQR, or the variance/standard deviation. The standard deviation (also variance) provides the most information, since it uses all of the values in the distribution in its calculation.

Fini

$\underline{\Sigma\quad}$

$\bar{Y} = \frac{\sum fY}{N}$ $\bar{Y} = \frac{\sum fY}{N}$ $\bar{Y} = \frac{\sum fY}{N}$

Cumulative Percentage Distribution

Minimum Age	Frequency	Percentage	Cumulative Percentage
14	1	3.7	3.7
15	1	3.7	7.4
16	9	33.3	40.7
17	4	14.8	55.5
18	12	44.4	99.9*
Total N	27	99.9*	

* Doesn't total to 100% due to rounding

Percentage Change in the Nursing Home Population, 1980-1990

Nine Regions of U.S.	Percentage	$Y - \bar{Y}$
Pacific	15.7	15.7 - 31.5 = -15.8
West North Central	16.2	16.2 - 31.5 = -15.3
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East North Central	23.2	23.2 - 31.5 = - 8.3
West South Central	24.3	24.3 - 31.5 = - 7.2
Middle Atlantic	28.5	28.5 - 31.5 = - 3.0
East South Central	38.0	38.0 - 31.5 = 6.5
Mountain	47.9	47.9 - 31.5 = 16.4
South Atlantic	71.7	71.7 - 31.5 = 40.2
(mean = 31.5)	$\sum Y =$ 283.1	$\sum (Y - \bar{Y}) = 0$

- One solution would be to take the absolute value for each number (ignore the minus signs). Unfortunately, absolute values are very difficult to work with mathematically.
- Fortunately, there is another alternative.